

# Year 9 Maths Knowledge Organiser

Foundation and Higher

# Algebra Foundation

## Solving linear equations

$$6x - 5 = 2x + 15$$

$$\begin{array}{r} -2x \\ -2x \end{array}$$

Subtract 2x from both sides

$$4x - 5 = 15$$

$$\begin{array}{r} +5 \\ +5 \end{array}$$

Add 5 to both sides

$$4x = 20$$

$$x = 5$$

Divide both sides by 4

## Factorising

Factorise  $x^2 + x - 42$

Sum of (+) 1  
-6 + 7 = 1

product of -42  
-6 x 7 = -42

$$x^2 + x - 42 = (x - 6)(x + 7)$$

## Expanding Brackets

$$(5x - 3)(2x + 1)$$

x	5x	-3
2x	10x <sup>2</sup>	-6x
+1	+5x	-3

$$= 10x^2 - 6x + 5x - 3$$

$$= 10x^2 - x - 3$$

Don't forget to simplify  
-6x + 5x = -x

## Rearranging formulae

$$y = ax + c$$

$$ax + c = y$$

$$ax = y - c$$

$$x = \frac{y - c}{a}$$

The 'aim' is to get 'x' on its own (x = ...)

## Algebra – Keywords.

- Substitution – replacing a letter with a number. (Letters next to each other means to multiply.)

Eg. Work out  $2g + 3h$  when  $g=9$  and  $h=4$

$$2 \times 9 + 3 \times 4$$

$$18 + 12$$

$$\underline{30}$$

- Expression – an algebraic sentence without an equal sign. (You may need to simplify but not solve.)

- Simplify – to make an expression have less terms.

Eg.  $2a + b + 3a + 2b - a - 2b = 4a + b$

Eg.  $5 \times r \times 2 \times p = 10rp$

- Solve – to work out an answer using algebra, to get  $x = \dots$

- Term – one part of an expression separated by a + or -

Eg.  $6p + 5q + 2r^2$  — 3 different terms.

- Expand – multiply to get rid of brackets.

Eg.  $3(x + 2) = 3x + 6$

- Factorise – opposite of expand, divide and put in brackets.

Eg.  $3x + 6 = 3(x + 2)$   
 $4x + 8 = 4(x + 2)$

- \* Indices an algebraic term that has a power Eg.  $3t^6$

- \* Sequence – an algebraic pattern going up by the same amount each time. In year 8 work out the formula called the nth term

- \* 'make x the subject' – re-arrange formula until it is in the form  $x =$

# Algebra Higher

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$$= 10x^2 - x - 3$$

Don't forget to simplify  
-6x + 5x = -x

a

$$y = ax + c$$

$$ax + c = y$$

$$ax = y - c$$

$$x = \frac{y-c}{a}$$

The 'aim' is to get 'x' on its own (x = ...)

b

$$ax - b = cx + b$$

$$ax - cx = 2b$$

$$x(a - c) = 2b$$

$$x = \frac{2b}{a-c}$$

Rearrange so that all the terms involving 'x' are on the same side

Divide by (a - c) to leave 'x' on its own

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  - \* Indices an algebraic term that has a power Eg.  $3t^4$
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# Calculation Foundation

## Ratio

Divide £48 in the ratio 3 : 5

3 : 5    8 parts in the ratio (3 + 5)  
 $£48 \div 8 = £6$     1 part = £6

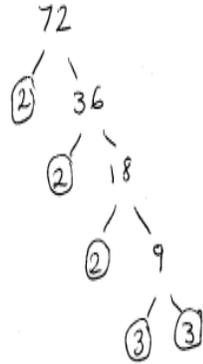
3 parts =  $3 \times £6 = £18$   
 5 parts =  $5 \times £6 = £30$

£18, £30

## Prime Factor

### Form

Write 72 as a product of its prime factors.



$$2 \times 2 \times 2 \times 3 \times 3$$

$$\text{or } 2^3 \times 3^2$$

$$2^3 \times 3^2$$

## Calculation – Key Words

- Integer - a whole Number
- Fraction - a part of a number has a numerator on the top and a denominator on the bottom
- Equivalent Fraction - two fractions which have the same value but are written differently-  
 $\frac{1}{2} = \frac{4}{8}$
- Percent - means out of 100, symbol %
- Multiple - any number in your original times table
- Factor - a number that goes into another number with no remainder
- Highest Common Factor - the biggest number that goes into two numbers - HCF of 12 and 16 is 4
- Lowest Common Multiple - the first number that appears in the times table of 2 different numbers - LCM of 3 and 5 is 15
- Prime Number - a number with only 2 factors, itself and 1
- Square number - the answer to a number multiplied by itself  
 $1 \times 1 = 1$      $2 \times 2 = 4$      $3 \times 3 = 9$
- Cube Number - the answer to a number multiplied by itself twice  
 $1 \times 1 \times 1 = 1$      $2 \times 2 \times 2 = 8$      $3 \times 3 \times 3 = 27$
- Square Root - Opposite of square number. This is the answer to what number multiplied by itself is the square number -  $\sqrt{16} = 4 \times 4$  so square root of 16 is 4
- Product means to multiply
- Sum means to add
- Share means to divide
- Difference means to subtract
- Evaluate - work out the answer
- Ratio - is comparing one quantity against another, written as a : b
- Significant figures - Is rounding to the most important (biggest value) digit - 2567 to 1 sf is 3000
- Standard Form - Is a method of writing very large or very small numbers -  $a \times 10^n$ . Where a is bigger than 1 and smaller than 10
- BIDMAS - Gives the order in which a calculation should be done
- Recurring - A decimal that continues forever with the same number after the decimal point
- Prime Factor - A factor of a number that is also a Prime number.
- Mixed Fraction - A fraction that has a whole number and a fraction
- Top heavy Fraction - A fraction where the numerator is bigger than the denominator

## Percentages

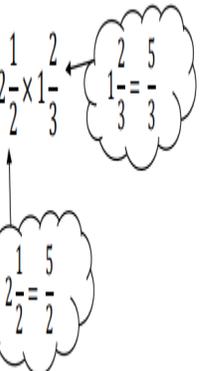
Decrease £350 by 10%

10% of £350 = £35 (divide by 10)

Decrease by 10% so take the 10% away from the original amount

$$£350 - £35 = £315$$

## Fractions



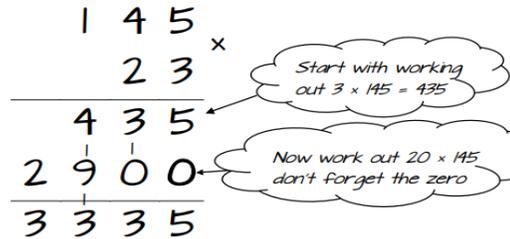
$$\frac{2}{2} \times 1 \frac{2}{3}$$

$$= \frac{5}{2} \times \frac{5}{3}$$

$$= \frac{25}{6} = 4 \frac{1}{6}$$

Change to a mixed number

## 145 x 23 = Multiplication



$$145 \times 23 =$$

	100	40	5
20	2000	800	100
3	300	120	15

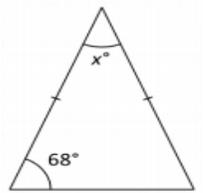
$$2000 + 800 + 100 +$$

$$300 + 120 + 15 = 3335$$



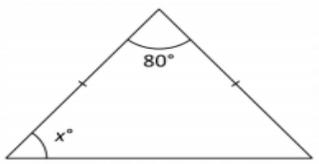
# Geometry Foundation

$$\begin{aligned}
 x^\circ + 68^\circ + 68^\circ &= 180^\circ \\
 x^\circ + 136^\circ &= 180^\circ \\
 x^\circ &= 180^\circ - 136^\circ \\
 x^\circ &= 44^\circ
 \end{aligned}$$



**In an isosceles triangle 2 of the angles are the same size**

$$\begin{aligned}
 x^\circ + x^\circ + 80^\circ &= 180^\circ \\
 2x^\circ + 80^\circ &= 180^\circ \\
 2x^\circ &= 100^\circ \\
 x^\circ &= 50^\circ
 \end{aligned}$$



## Parallel lines

Alternate angles are equal

Corresponding angles are equal

Allied (or co-Interior) angles add up to 180

## Polygons

$a + b + c + d + e + f = 360$

Exterior angles of a polygon add up to 360

$a + b = 180$

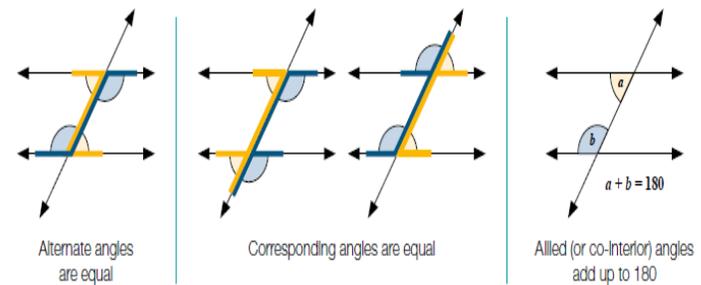
The Interior and exterior angle of any polygon add up to 180

## Geometry Key Words and Formula's

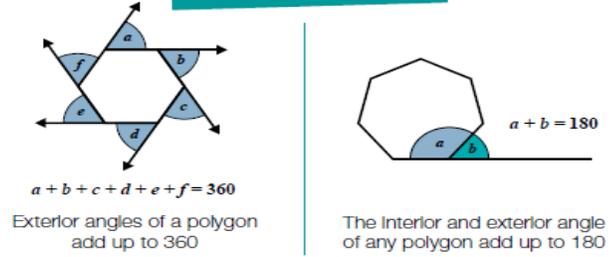
- \* Area – the space inside a 2D shape measured in units 2
- \* Perimeter - the distance around the outside of a shape (called circumference for circles)
- \* Volume – the space inside a 3D shape
- \* Surface Area – the area of the flat faces of a 3D shape
- \* Angle - The space made when two lines meet, measured in degrees
- \* Acute angle – less than  $90^\circ$ , obtuse angle bigger than  $90^\circ$  smaller than  $180^\circ$ . Straight line angle equal to  $180^\circ$ . Reflex angle bigger than  $180^\circ$  but smaller than  $360^\circ$ .
- \* Angles in a straight line add to  $180^\circ$
- \* Angles in a triangle add to  $180^\circ$
- \* Angles around a point add to  $360^\circ$
- \* Parallel lines – these lines have the same gradient and they never meet
- \* Perpendicular lines – these lines cross at  $90^\circ$
- \* Alternate angles – these two angles are the same in parallel lines (Z angle)
- \* Corresponding angles – these two angles are the same in parallel lines (F angles)
- \* Co – Interior angles – these two angles add up to  $180^\circ$  (C angles)
- \* Scalene triangle – A triangle with three different sides and three different angles
- \* Isosceles triangles – A triangle that has the two sides the same length and the base angles the same
- \* Equilateral triangle – A triangle that has three sides the same and three angles the same
- \* Polygon - A 2D shape that has only straight sides (edges)
- \* Interior and Exterior angles – The exterior angles of any polygon always add to  $360^\circ$ . The interior angles + exterior angles always add to  $180^\circ$
- \* Bearing - A bearing is an angle that starts from North, goes **clockwise** and must have three figures in it
- \* Reflection – is flipping a shape over a mirror line
- \* Rotation – is spinning a shape from a certain point, the direction of movement is required
- \* Translation – is pushing a shape horizontally and vertically. It is written as a vector
- \* Enlargement – is making a shape bigger or smaller by a scale factor from a centre point

# Geometry Higher

## Parallel lines



## Polygons



## Geometry Key Words and Formula's

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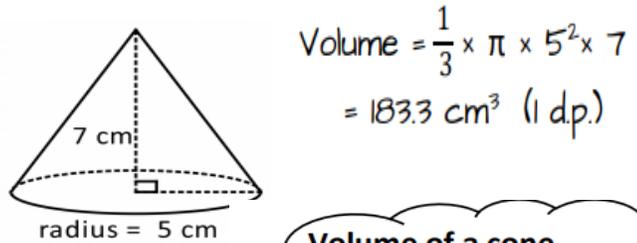
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## Volumes

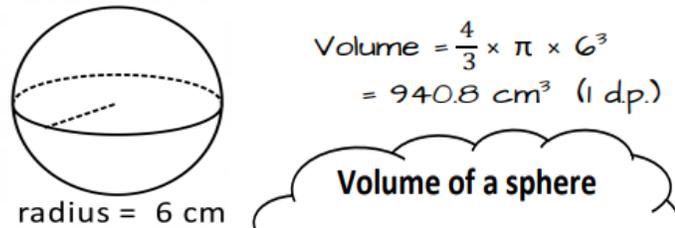
Cuboid = $l \times w \times h$	
Prism = area of cross section $\times$ length	
Cylinder = $\pi r^2 h$	
Volume of pyramid = $\frac{1}{3} \times$ area of base $\times$ h	

Calculate the volume



**Volume of a cone**  
 $\frac{1}{3} \times \pi \times \text{radius}^2 \times \text{height}$

Calculate the volume



**Volume of a sphere**  
 $\frac{4}{3} \times \pi \times \text{radius}^3$

# Statistics Foundation

## AVERAGES

The **MODE** is the value that occurs most often

② 4, ② 6, 1, 4, ②

There are more 2's in the data list than any other number so the **MODE** = 2

The **MEDIAN** is the value in the middle when the data list listed in order

Example 1

2, 5, 4, 2, 1

Write the data in order

1, 2, 2, 4, 5

MEDIAN = 2

Example 2

4, 5, ⑤, ⑧, 10

If there are 2 numbers in the middle

$$\text{MEDIAN} = \frac{5+8}{2} = \frac{13}{2} = 6.5$$

To calculate the **MEAN**

3, 6, 3, 10, 8

Step 1: Add all of the 'numbers' together

$$3 + 6 + 3 + 10 + 8 = 30$$

Step 2: Divide your total by the number of values

$$\begin{aligned} \text{in your data list} \quad \text{MEAN} &= 30 \div 5 \\ &= 6 \end{aligned}$$

The **RANGE** is the difference between the largest and smallest number in the list  $10 - 3 = 7$

### Statistics Key Words

Range – the difference between the largest and the smallest number from a list of numbers

Mode – The number that appears the most often from a list of numbers

Median – the number in the middle of a list of ordered numbers

Mean – Add all the numbers up and then divide this total by the amount of numbers that were there

Averages – Mean, Median and Mode. Three averages to help determine common or a representative number from a list of numbers

Pie Charts – A way of representing data in a circle. All pie charts add up to 360°.

Probability - the chance of an event happening. Probability must be written as a fraction, decimal or a percentage. Not as a ratio.

Tree Diagrams – A diagram to show the probability of two or more events happening

Probability 'OR' Rule – The Probability of event A or event B happening is  $P(A) + P(B)$

Probability 'AND' Rule – The Probability of event A and event happening is  $P(A) \times P(B)$

Probability – of all possible events adds to 1

Venn Diagrams – Uses two circles often overlapping to show data

Scatter Diagram – A graph that shows the relationship between two variables

Correlation – Used to describe the relationship in scatter diagrams – positive both go up or down, negative – one goes up as the other goes down, no correlation – there is no link between the two variables

Line of best Fit – A straight line drawn through the scatter diagram with roughly half the data points on either side of the line

Frequency Table – Data is put into groups in a table. Used to help find averages

Pictogram – Is a way of showing data using pictures. It must have a key explaining what the picture stands for

Stem and Leaf – Shows numbers in a table, the leaf is the last digit of the piece of data. Needs a key

## Pictogram

The pictogram shows information about the number of books sold by an author in January, February and March.

January	
February	
March	
April	

Key:	
	represents 8 books

(a) Write down the number of books sold in March

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} = 8 \quad \square = 2$$

14 books were sold in April.

(b) Show this information on the pictogram.

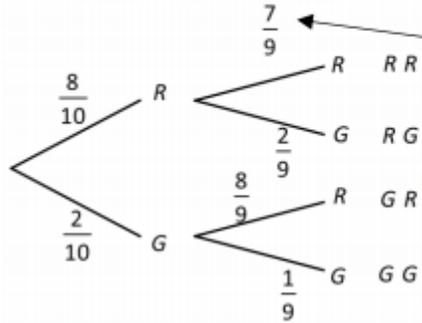
$$\begin{array}{r} 26 \\ \hline (1) \end{array}$$

(1)

# Statistics Higher

## Probability

A bag contains 8 red and 2 blue balls. Two balls are selected at random



Take care with the probabilities for the second pick - now 9 balls left in the bag if a red picked out first then there are only 7 red balls left for the second pick.

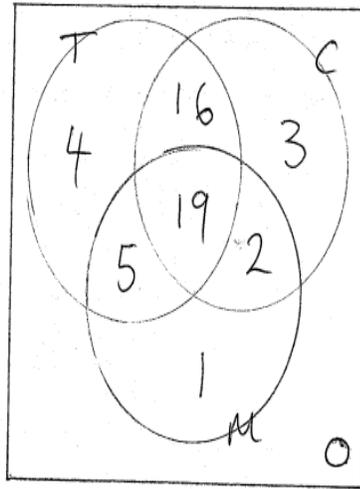
## Venn Diagram

Sami asked 50 people which drinks they liked from tea, coffee and milk.

- All 50 people like at least one of the drinks
- 19 people like all three drinks
- 16 people like tea and coffee but do not like milk.
- 21 people like coffee and milk.
- 24 people like tea and milk.
- 40 people like coffee.
- 1 person likes only milk.

Sami selects at random one of the 50 people.

Work out the probability that this person likes tea.



$$\frac{44}{50}$$

## Mean from a table

Adam is measuring the heights in cm of his tomato plants.

Height (cm)	<i>m.p</i>	Frequency	<i>m.p x f</i>
$140 < h \leq 150$	145	7	1015
$150 < h \leq 160$	155	10	1550
$160 < h \leq 170$	165	15	2475
$170 < h \leq 180$	175	19	3325
$180 < h \leq 200$	190	9	1710
		60	10075

(a) Estimate the mean height.  
Give your answer correct to 1 decimal place.

$$\frac{10075}{60} = 167.9 \text{ (1dp)}$$

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Stem and Leaf – Shows numbers in a table, the leaf is the last digit of the piece of data. Needs a key

Histogram – A diagram that uses the area of a section to represent the frequency, not the height. It looks like a bar graph with bars of different widths

Cumulative Frequency - A diagram that can be used to find the median from the curve on the graph. Cumulative frequency, means the frequency added up

Box Plots – Uses a box which contains the middle 50% of the data. Diagram has Lowest Value, Lower Quartile, Median, Upper Quartile and Highest Value

Lower and Upper Quartiles- these are 25% and 75% of the way through the ordered data

Inter-Quartile Range – Upper Quartile – Lower Quartile. It is the middle 50% of the data